A coupled boundary layer mixing length model for gas transfer at the air-water interface

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Coupled boundary modeling with mixing lengths

- Modeling the average properties of the boundary layers at the air-water interface requires closure of the Reynolds equations.

- The simplest closure represents the diffusivity of the turbulence in analogy to molecular viscosity:

\[
(v + v_t) \frac{dU}{dz} = u_*^2
\]

- Where constant flux layers on both sides of the interface are assumed.
- In describing the turbulent boundary layer, the Eddy Viscosity is taken to be

\[
v_t = \kappa u_* z
\]

- Leading to the usual logarithmic “law of the wall”.
In the context of gas transfer this is not adequate as the main resistance to gas transfer comes from the thin diffusive layers on either side of the interface.

The concept of mixing length is particularly useful as it provides a physical picture of the eddy sizes that are dominant in the mixing process. It may be thought of the average vertical distance eddies travel before losing their characteristics (velocity, temperature, etc.) to the average surrounding flow. Formally, it is related to the Eddy Viscosity:

\[ \nu_t = l_u^2 \frac{dU}{dz} \]

where in the logarithmic layer:

\[ l_u = \kappa z \]
Clearly, there is a need to have a smooth transition between the viscous sub-layer, having a linear profile, and the logarithmic layer. This was suggested by van Driest (1956) and used to good effect in several descriptions of boundary layers on smooth solid walls:

\[ l_u = \kappa z \left[ 1 - \exp\left(-zu_* / 13\nu\right) \right]^2 \]
The mixing length model

normalized coupled profiles of wind and current in smooth flow

\[ \text{air-water interface} \]
To deal with flows that are rough or transitional – those in which the thickness of the viscous boundary layer is less than the height of the roughness elements – Kitaigorodskii and Donelan (1984, in the first of these symposia) introduced a non-zero mixing length at the surface,:

\[ l_u = l_{us} + \kappa z [1 - \exp(-zu_*/13\nu)]^2 \]

and the corresponding mixing length for passive scalars:

\[ l_g = Sc_t \frac{1}{2} (l_{gs} + \kappa z [1 - \exp(-zu_*/13\nu)]^2) \]
Where $Sc_t$ is the turbulent Schmidt number and is a function of the Monin-Obukhov stability index $\zeta = z/L_O$.

$$L_O = -\frac{u_*^3}{\kappa a g Q_o/(c_p \rho)} ,$$

Thus the universal behavior of diabatic boundary layers is built in to the mixing length.

The ratio of surface mixing lengths for gases to that for momentum is necessarily a function of the other relevant non-dimensional numbers, i.e. the molecular Schmidt number and the roughness Reynolds number:

$$l_{gs} = aSc^m Re^n l_{us}$$

where the empirical coefficients are set by comparison with the data of Moller and Schumann (JGR, 1970), Liss (DSR, 1973) and Ocampo-Torres et al., (Tellus, 1994).
Mixing length model compared to the data of Moller and Schumann, JGR, 1970.

\[
\frac{V}{U_x} = \frac{C_g}{\sqrt{C_f}}
\]
Mixing length model compared to the evaporation data of Liss, DSR, 1973.
mass transfer velocity for H₂O. Tank experiment of Ocampo-Torres et al., Tellus, 1994
normalized mass transfer velocity versus Schmidt # for smooth and rough flow in air

Smooth = blue, asymptotic slope = -0.82
Rough = red, asymptotic slope = -0.67
Wave breaking and enhancement of gas transfer of water-side limited gases

- Wave breaking is a source of vertical turbulent energy that acts to disrupt the thin viscous sub-layer near the surface. It can be considered in the context of Monin-Obukhov similarity theory to have much the same role as buoyancy does in being a source of vertical turbulence that effectively mixes the boundary layer. The M-O index is thus computed from the distribution of wave dissipation with depth due to waves of all wavelengths.

\[ L_{om} = -\frac{u_*^3}{\kappa \alpha_T g Q_o / (c_p \rho) + \kappa \varepsilon_w} \approx -\frac{u_*^3}{\kappa \varepsilon_w} \]

- This produces an enhancement of the mass transfer velocity as shown. The enhancement is greater for the tank data than the oceanic data because the short waves in a tank distribute their dissipating energy over smaller depths.
Dissipation Data
normalized mass transfer velocity versus Schmidt # for smooth and rough flow in water

Smooth = blue
Rough (not breaking) = red
Rough (breaking, Tank at 10 m) = green
Rough (breaking, Ocean at 100 km) = magenta
Conclusions

- The coupled mixing length model provides a means of calculating mass transfer velocities for ‘contaminants’ that are limited in either phase as well as those that are limited in both sublayers.

- The various empirical coefficients that define the relationship between mixing lengths for momentum and passive scalars were set by comparing with the wide Schmidt number range data of Moller and Schumann for Schmidt number dependence and to the evaporation data of Liss and of Ocampo-Torres et al. for the roughness Reynolds number dependence.

- The hydrostatic stability effects in the air boundary are readily included using the usual Monin-Obukhov similarity relations.

- The model, thus established for the air boundary layer, is applied to the water boundary layer with the important role of wave breaking in mixing the sub-layer included through the Monin-Obukhov index computed from the ratio of vertical turbulent energy introduced by wave breaking to the energy extracted from the (horizontal) mean flow.